Topics in Rational and Integral Points

Week 2 (September 9 – 13, 2019)

Venue: Lecture hall -101 in Old University, Rheinsprung 9, Basel

	Monday	Tuesday	Wednesday	Thursday	Friday
08.30 - 09.15	*Registration*				
09.15 - 10.05	**Capuano**	**Destagnol**	**Pasten**	**Jones**	**Binyamini**
10.05 - 10.45	*Coffee break*				
10.45 - 11.35	**Zannier**	**Salberger**	**Rémond**	**Pila**	**Barroero**
11.50 - 12.40	**Pazuki**	**Browning**	**Peyre**	**Pieropan**	**Bilu**
12.40 - 14.30	*Lunch*			*Lunch*	
14.30 - 15.20	**Krieger**	**Gao**		**Ho**	
15.20 - 16.00	*Coffee*			*Coffee*	
16.00 - 16.50	**Viada**	**Ullmo**		**Widmer**	
18.00 -	*Welcome apéro*				

Fabrizio Barroero   Friday, 10.45 - 11.35
On the Zilber-Pink Conjecture for complex abelian varieties

Abstract: The Zilber-Pink conjecture roughly says that the intersection of a subvariety of an abelian variety with its algebraic subgroups of large enough codimension is well behaved. In the case the subvariety has dimension 1, if the abelian variety and the subvariety are defined over the algebraic numbers, Habegger and Pila proved the conjecture, thus showing that the intersection of a curve with algebraic subgroups of codimension at least 2 is finite, unless the curve is contained in a proper algebraic subgroup. Together with Gabriel Dill, using a recent result of Gao, we extended this statement to complex abelian varieties. More generally, we showed that the whole conjecture for complex abelian varieties can be deduced from the algebraic case.

Yuri Bilu   Friday, 11.50 - 12.40
Trinomials with given roots

Abstract: We show that, apart from some obvious exceptions, the number of trinomials vanishing at given complex numbers is bounded by an absolute constant. When the numbers are algebraic, we also bound effectively the degrees and the heights of these trinomials. A joint work with Florian Luca.

Gal Binyamini   Friday, 09.15 - 10.05
Point counting for foliations over number fields

Abstract: In previous work with Novikov we established the Wilkie conjecture for sets defined using holomorphic Pfaffian functions on compact domains. This applies to elliptic and abelian functions, but not to modular functions and more general functions required in diophantine applications of the Pila-Wilkie theorem around Shimura varieties and abelian families. In this talk I will discuss an alternative approach for general foliations over number fields, where the Pfaffian methods are replaced by Nevanlinna-type arguments. We show first, that if the foliation satisfies a certain diophantine tameness condition then one can obtain an effective polylogarithmic counting theorem; and second, that this tameness condition is closely related to functional transcendence and holds for many of the foliations involved in the applications of the counting theorem around Shimura varieties and abelian families. Consequently this new approach greatly increases the scope of applications compared to our previous methods. If time permits I'll also discuss some consequences of this counting theorem in diophantine geometry.

Tim Browning   Tuesday, 11.50 - 12.40
Free rational points on hypersurfaces

Abstract: Motivated by a recent question of Peyre, we apply the Hardy-Littlewood circle method to count "sufficiently free" rational points of bounded height on smooth projective hypersurfaces of low degree that are defined over the rationals.

Laura Capuano   Monday, 09.15 - 10.05
Lang-Vojta conjecture over function fields for surfaces dominating tori

Abstract: The celebrated Lang-Vojta Conjecture predicts degeneracy of S-integral points on varieties of log general type defined over number fields. It admits a geometric analogue over function fields, where stronger results have been obtained applying a method developed by Corvaja and Zannier. In this talk, we present a recent result for non-isotrivial surfaces over function fields dominating a two-dimensional torus. This extends Corvaja and Zanniers result in the isotrivial case and it is based on a refinement of gcd estimates for polynomials evaluated at S-units. This is a joint work with A. Turchet.

Kevin Destagnol   Tuesday, 09.15 - 10.05
Counting points of given degree via the height zeta function

Abstract: Let $X = \mathrm{Sym}^d \mathbf{P}^n := \mathbf{P}^n \times \cdots \times \mathbf{P}^n / \mathfrak{S}_d$ where the symmetric d-group acts by permuting the d copies of $\mathbf{P}^n$. Manin's conjecture gives a precise prediction for the number of rational points on X of bounded height in terms of geometric invariants of a resolution of X and the study of Manin's conjecture for X can be derived from the geometry of numbers in the cases $n > d$ and for $n = d = 2$. In this talk, I will explain how one can use the fact that $\mathbf{P}^n$ is an equivariant compactification of an algebraic group and the height zeta function machinery in order to study the rational points of bounded height on X in new cases that are not covered by the geometry of numbers techniques. This might in particular be an interesting testing ground for the latest refinements of Manin's conjecture.

Wei Ho   Thursday, 14.30 - 15.20
The Hasse principle for some genus one curves

Abstract: We will discuss problems related to the Hasse principle for certain genus one curves, namely those given by bihomogeneous polynomials of bidegree (2,2) in $\mathbb{P}^1 \times \mathbb{P}^1$. In particular, we compute the proportion of these curves that are everywhere locally soluble (joint work with Tom Fisher and Jennifer Park), and we show that the Hasse principle fails for a positive proportion of these curves, by comparing the average sizes of 2- and 3-Selmer groups for a family of elliptic curves with a marked point (joint work with Manjul Bhargava).

Gareth Jones   Thursday, 09.15 - 10.05
Effective relative Manin-Mumford for families of multiplicative extensions of an elliptic curve

Abstract: I will discuss joint work with Harry Schmidt in which we give an effective version of a result of Bertrand, Masser, Pillay and Zannier on families of multiplicative extensions of an elliptic curve. In certain cases we obtain extra uniformity. The methods involve pfaffian functions. In particular, previous work with Schmidt on pfaffian definitions of elliptic functions plays a key role.

Holly Krieger   Monday, 14.30 - 15.20
Uniform Manin-Mumford and Bogomolov bounds in genus 2

Abstract: I will discuss joint work with Laura DeMarco and Hexi Ye in which we introduce a general strategy for quantitative bounds on points of small height. We apply this strategy to prove a uniform Manin-Mumford bound over the complex numbers, and a uniform Bogomolov bound over the algebraic numbers, for a two-dimensional family of genus 2 curves. Time permitting, I will discuss ideas for making these bounds effective.

Hector Pasten Wednesday, 09.15 - 10.05
Towards Hilbert's tenth problem for rings of integers

Abstract: Hilbert's tenth problem (H10) asked for an algorithm to decide solvability of diophantine equations over the integers. By results of Davis, Putnam, Robinson and Matiyasevich, the requested algorithm does not exist. The analogue of H10 for rings of integers of number fields remains open in general. Using quadratic twists to control Selmer ranks, Mazur and Rubin showed that a negative solution would follow from a squareness conjecture for Shafarevich-Tate groups of elliptic curves over number fields. In this talk I will explain how a negative solution for H10 of rings of integers would also follow from modularity of elliptic curves over number fields and the rank part of the BSD conjecture (joint work with Ram Murty). I will also present a recent unconditional construction of new families of number fields whose rings of integers have a negative solution of H10, using Iwasawa theory and congruences of Heegner points (joint work with Natalia Garcia-Fritz).

Fabien Pazuki Monday, 11.50 - 12.40
Regulators of elliptic curves over global fields

Abstract: In a recent collaboration with Pascal Autissier and Marc Hindry, we prove that up to isomorphisms, there are at most finitely many elliptic curves defined over a fixed number field, with Mordell-Weil rank and regulator bounded from above, and rank at least 4. We will explain how to obtain an even stronger result in the case of elliptic curves defined over a function field of characteristic $p > 0$, in particular removing the conditions on the rank (while adding a necessary assumption on the inseparability degree).

Emmanuel Peyre Wednesday, 11.50 - 12.40
Progress on Manin's program

Abstract: In recent years the comprehension of the distribution of points of bounded height has known significant progress. In particular, the expected behaviour was proven on the complement of accumulating thin subsets in several cases and the work of S. Tanimoto, B. Lehmann and A. Sengupta gave a better understanding of the thin subsets on Fano varieties. At the same time, new questions were raised regarding extensions of the program, beyond Fano varieties, and when using other invariants. This talk will give a survey of the state of the art in this program.

Marta Pieropan Thursday, 11.50 - 12.40
On the distribution of Campana points on Fano varieties

Abstract: We call Campana points an arithmetic notion of points on Campana's orbifolds that has been first studied by Campana and Abramovich, and that interpolates between the notions of rational and integral points. In this talk we introduce Campana points and a Manin type conjecture for Campana points on Fano varieties, and we present results for equivariant compactifications of vector groups (joint work with A. Smeets, S. Tanimoto, T. Várilly-Alvarado) and for toric varieties (joint work with D. Schindler).

Jonathan Pila   Thursday, 10.45 - 11.35
Independence of CM points in elliptic curves

Abstract: I will speak about joint work with Jacob Tsimerman. Let E be an elliptic curve parameterized by a modular (or Shimura) curve. There are a number of results (..., Buium-Poonen, Kühne) to the effect that the images of special points tend to be linearly independent in E. We consider this issue in the setting of the Zilber-Pink conjecture and prove a result which improves previous results in some aspects.

Gaël Rémond   Wednesday, 10.45 - 11.35
Properties of the Faltings height

Abstract: We prove an inequality for the Faltings heights of abelian varieties in a short exact sequence. This is equivalent to a kind of parallelogram inequality for the Faltings heights of certain isogenous abelian varieties. The proof relies on a formula, essentially due to Raynaud, expressing the variation of the Faltings height by an isogeny in terms of rational points. We give an application to abelian varieties of minimal height in an isogeny class.

Per Salberger   Tuesday, 10.45 - 11.35
On the density of rational points and the covering gonality of hypersurfaces

Abstract: I will present new uniform estimates for the number of rational points of bounded height on non-singular threefolds in $\mathbf{P}^4$. These are obtained by means of a global version of Heath-Brown's p-adic determinant method and recent geometric results on covering gonality. As an application we obtain new estimates for equal sums of three powers, which are much stronger than the previous estimates of Hua, Browning/Heath-Brown and myself.

Emmanuel Ullmo   Tuesday, 16.00 - 16.50
Homogeneous Measures on Satake Compactifications of Locally Symmetric Spaces

Abstract: We prove that the set of homogeneous probability measures on the maximal Satake compactification of a locally symmetric space $\Gamma\backslash G/K$ is compact. We give some applications to the equidistribution of sequences of weakly special subvarieties of Shimura varieties. This is a joint work with Christopher Daw and Alexander Gorodnik.

Evelina Viada   Monday, 16.00 - 16.50
Transversality and rational points.

Abstract: The transversality of a curve is a crucial condition in many theorem in the context of anomalous intersections. However, given a family of curves in E^n for E an elliptic curve, it is not easy to decide weather they are transverse of not. I will present a criterium and some examples of families of transverse curves. Then, using some generalisations of bounds for height of points of small rank on a transverse curve, I will show how to determine the rational points or K-rational points on such families.

Martin Widmer   Thursday, 16.00 - 16.50

Averages for the ℓ-torsion in class groups

Abstract: We present new upper bounds for the average size of the ℓ-torsion $\mathrm{Cl}_K[\ell]$ of the class group of K, as K runs through certain natural families of number fields and ℓ is a positive integer. Our results use a refinement of a key argument, used in most results of this type, which links upper bounds for $\mathrm{Cl}_K[\ell]$ to the existence of many small primes splitting completely in K. The improvements are achieved by replacing the discriminant in this key argument with a new family of specialised invariants of number fields, in conjunction with new counting results for these invariants. This is joint work with Christopher Frei.

Umberto Zannier   Monday, 10.45 - 11.35

Bounded height in pencils of finitely generated subgroups

Abstract: The context is about sections of a pencil of tori or abelian varieties. Several results concern the set of parameters which destroy independence. We shall look at a generalization of this issue: "For which parameters does a finitely generated group of sections meet a given sub-variety ?" We shall discuss some joint results with Amoroso and Masser, for toric pencils: under suitable assumptions we prove bounded height for the said parameters. There are diophantine applications.